Guide to the New York State Common Core Standards

ALGEBRA II

Revised November 1, 2017

INTRODUCTION

This document is designed to be a comprehensive resource for New York State Common Core Standards for Algebra II, released in 2015. It includes information compiled from various additional documents published by the New York State Education Department (NYSED) on the Common Core standards, including clarifications published after the standards were released. It also contains an unofficial interpretation of levels of knowledge proficiency for each standard. This document contains the following:

- Overview of all New York State Common Core standards (p. 3), which includes a summary of all standards and an approximate percentage breakdown on the Regents Exam for each of the major conceptual categories for the course:
  - Number and Quantity: 5-12% of Regents Exam
  - Algebra: 35-44% of Regents Exam
  - Functions: 30-40% of Regents Exam
  - Statistics and Probability: 14-21% of Regents Exam
- **STANDARD:** Complete text of each standard, grouped by cluster and domain
- **EMPHASIS:** Level of emphasis for each cluster in the course, as stipulated by NYSED:

■■■ Major content (51-65% of Regents)	■■□ Supporting content (14-28% of Regents)	■□□ Additional content (19-33% of Regents)

- **NOTES:** Clarifications for standards, written by NYSED or PARCC (all NYSED exams will follow the framework articulated by PARCC[1])
- **RELATED:** Standards that cover similar or related topics
- **LEVELS:** Unofficial interpretation of levels of knowledge proficiency for each standard, ranging from most proficient to least proficient. These levels are designed to provide guidance on how each standard could be taught. This represents my interpretation of the levels, not NYSED's or anyone else's, although some of the levels are based on NYSED's performance level descriptions (http://www.engageny.org/resource/performance-level-descriptions-for-ela-and-mathematics), which are used in the process of creating Regents Exams.
  - **Level 5:** Knowledge that exceeds what is required to meet the standard
  - **Level 4:** Minimum knowledge required to meet the standard
  - **Level 3:** Approaching minimum knowledge required to meet the standard
  - **Level 2:** Developing level of knowledge required to meet the standard, such as a direct application of a formula or theorem
  - **Level 1:** Prerequisite skills, such as definitions, needed in order to learn the standard
- **EXAMPLES:** Examples of each level for each standard. (These are illustrations of each standard, not necessarily the only possible questions for each level) Many of the examples are taken from previous Regents Exams.

Updated versions of this file and guides to New York State standards for other high school mathematics courses will be posted online at http://www.reachthesource.org .

Bobson Wong
Mathematics Teacher, New York City

SOURCES

Algebra II (Common Core) Regents High School Examination, various dates, University of the State of New York.

JMAP Regents by Common Core State Standard: Topic, http://www.jmap.org/JMAP_REGENTS_BOOKS.htm.

Math A Regents High School Examination, various dates, University of the State of New York.

New York State Common Core Algebra II Standards Clarifications, http://www.engageny.org/resource/regents-exams-mathematics-algebra-ii-standards-clarifications.

New York State Common Core Mathematics Curriculum, EngageNY, http://www.engageny.org/resource/high-school-algebra-ii.

New York State P-12 Common Core Learning Standards for Mathematics, http://www.engageny.org/resource/new-york-state-p-12-common-core-learning-standards-for-mathematics .

New York State Regents Examination in Algebra II (Common Core): Performance Level Descriptors, August 2014, http://www.engageny.org/resource/performance-level-descriptions-for-ela-and-mathematics.

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[1] http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf.

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OVERVIEW OF COMMON CORE ALGEBRA II STANDARDS

This overview contains all standards grouped by major conceptual categories for the course (Algebra, Functions, Number and Quantity, Statistics). Standards that are shared with Algebra I are labeled [I] below. Each standard is labeled below as Major content (51-65% of Regents) (≡), Supporting content (14-28% of Regents) (=), or Additional content (19-33% of Regents) (−) to indicate its emphasis in the course as specified by the New York State Education Department (NYSED).

ALGEBRA (35-44% OF REGENTS EXAM)

Polynomials

≡ Rewrite expressions in equivalent forms (A-SSE.A.2) [I]

≡ Write expressions in equivalent forms to reveal properties (A-SSE.B.3) [I]

≡ Apply the Remainder Theorem (A-APR.B.2)

≡ Identify zeroes of quadratic and cubic polynomials and use them to sketch graphs (A-APR.B.3) [I]

= Solve quadratic equations by inspection, taking square roots, factoring, completing the square, quadratic formula, graphing) (A-REI.B.4) [I]

= Solve quadratic equations with complex solutions (N-CN.C.7)

= Divide polynomials with remainder, incl. with long division (A-APR.D.6)

= Prove and use polynomial identities (A-APR.C.4)

Rational and Radical Equations

≡ Justify steps in solving rational or radical equations (A-REI.A.1b) [I]

≡ Solve rational and radical equations, identify extraneous solutions (A-REI.A.2)

Systems of Equations

− Solve systems of three linear equations in three variables (A-REI.C.6)

− Solve a quadratic-linear system of equations algebraically and graphically (A-REI.C.7)

≡ Approximate, justify, interpret graphical solution to $f(x) = g(x)$ (A-REI.D.11) [I]

Geometry

− Derive equation of parabola given focus and directrix (G-GPE.A.2)

Algebra and Modeling

= Create one-variable equations and inequalities (A-CED.A.1) [I]

STATISTICS (14-21% OF REGENTS EXAM)

Univariate and Bivariate Data

− Determine if a normal curve is appropriate for data. Determine population percentages using a normal distribution (S-ID.A.4)

= Represent bivariate data on scatterplot (S-ID.B.6) [I]

= Fit linear, quadratic, exponential functions to data (S-ID.B6a) [I]

Inference

≡ Determine if a statistic is likely to occur based on a given simulation (S-IC.A.2)

≡ Understand uses of, relationship of randomization to, and differences between surveys, experiments, observational studies (S-IC.B.3)

≡ Given simulation model based on sample, construct 95% interval centered on sample; determine if suggested parameter is plausible (S-IC.B.4)

≡ Compare two treatments and determine if the difference between parameters is significant (S-IC.B.5)

≡ Use statistical language to draw conclusions from numerical summaries (S-IC.B.6a) and critique claims (S-IC.B.6b)

Conditional Probability

− Describe events as subsets of sample space or unions, complements, intersections of other events (S-CP.A.1)

− Determine if events are independent (S-CP.A.2)

− Calculate and interpret conditional probability (S-CP.A.3, S-CP.B.6)

− Construct, interpret, use two-way tables to determine if events are independent (S-CP.A.4)

− Use Addition Rule of probability and interpret the answer (S-CP.B.7)

NUMBER & QUANTITY (5-12% OF REGENTS EXAM)

Rational Exponents

≡ Explore rational exponents as extension of integer exponents (N-RN.A.1)

≡ Convert between expressions with radicals and rational exponents (N-RN.A.2)

Complex Numbers

− Understand i and $a + bi$ form (N-CN.A.1)

− Add, subtract, multiply complex numbers (N-CN.A.2)

FUNCTIONS (30-40% OF REGENTS EXAM)

Properties of Functions

≡ Sketch graphs of functions given verbal description, interpret key features of graphs and tables (F-IF.B.4) [I]

≡ Calculate and interpret average rate of change of a function over an interval (F-IF.B.6) [I]

= Graph and show features of graphs (F-IF.C.7)

= Graph polynomial functions and show zeroes and end behavior (F-IF.C.7c)

= Graph cube root, exponential, log (show intercepts, end behavior), trig (show period, midline, amplitude) functions (F-IF.C.7e)

= Write a function in different forms to reveal its properties (F-IF.C.8) (e.g. Interpret exponential functions and classify as growth or decay) (F-IF.C.8b) [I]

= Compare properties of two functions represented in different ways (F-IF.C.9) [I]

− Find the inverse of a function (F-BF.B.4)

Exponential and Logarithmic Functions

≡ Rewrite exponential expressions (A-SSE.B.3c) [I]

= Use logarithms to solve exponential equations (base 2, 10, e), evaluate logs (F-LE.A.4)

Sequences and Series

= Construct linear and exponential functions, incl. arithmetic and geometric sequences (F-LE.A.2) [I]

= Identify explicit and recursive sequences as functions with integer domain (F-IF.A.3) [I]

≡ Write arithmetic and geometric sequences explicitly and recursively, translate between the forms, use for modeling (F-BF.A.2)

≡ Derive and use formula for geometric series with summation notation (A-SSE.B.4)

Trigonometric Functions

− Define radian measure (F-TF.A.1)

− Use unit circle and given angles in radian measure to calculate values of 6 trig functions (F-TF.A.2)

− Use sine or cosine functions to model periodic behavior (F-TF.B.5)

− Prove Pythagorean identity and use it to find trig functions given values of other trig functions (F-TF.C.8)

Functions and Modeling

≡ Write a function to describe a relationship (F-BF.A.1) [I]

≡ Combine functions using arithmetic operations (F-BF.A.1b)

− Transform functions, recognize even and odd functions (F-BF.B.3) [I]

− Interpret parameters of linear or exponential function in context (F-LE.B.5) [I]

NUMBER AND QUANTITY (5-12% OF REGENTS EXAM)

THE REAL NUMBER SYSTEM (N-RN)

A. Extend the properties of exponents to rational exponents.

(N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $\left(5^{\frac{1}{3}}\right)^{3}$ to equal 5.

Emphasis: ■■■ Major content (51-65% of Regents)
Related: Convert between expressions with radicals and rational exponents (N-RN.A.2)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain why two algebraic expressions containing radicals and rational exponents are equal.	Explain how $\left(x^{\frac{1}{3}}y^{2}\right)^{\frac{3}{2}}$ can be written as the equivalent radical expression $y^{3}\sqrt{x}$.
LEVEL 4 *(meets standard)*	Explain why two numerical expressions containing radicals and rational exponents are equal.	(Aug. 2016 Alg. II, #26) Explain how $\left(3^{\frac{1}{5}}\right)^{2}$ can be written as the equivalent radical expression $\sqrt[5]{9}$.
LEVEL 3	Justify each of the steps involved in rewriting a radical expression as an expression with a rational exponent.	Let $\sqrt{4}\sqrt{4}=4^{x}4^{x}$. State the appropriate rule of exponents that justifies each step: $4^{x}4^{x} = 4^{x+x}$ $4^{2x} = 4^{1}$ $2x = 1$ $x = 1/2$
LEVEL 2	Solve exponential equations with rational roots by rewriting each side of the equation with the same base.	Solve the following equations for the variable: $3^{2x} = 3^{1}$ $3^{2x} = 9^{1}$
LEVEL 1	Simplify powers with integer exponents.	If $x^{3}x^{5} = x^{a}$, find the value of a.

(N-RN.A.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Includes expressions with variable factors such as $\sqrt[3]{27x^5y^3}$ (NYSED).

Related: (N-RN.A.1)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain why two algebraic expressions containing radicals and rational exponents are equal.	Explain how $\left(x^{\frac{1}{3}}y^2\right)^{\frac{3}{2}}$ can be written as the equivalent radical expression $y^3\sqrt{x}$.
LEVEL 4 *(meets standard)*	Rewrite multivariable expressions and perform operations on expressions involving radicals and/or rational exponents.	Rewrite $\left(\sqrt[3]{27x^5y^3}\right)^6$ as an expression of the form $a^px^qy^r$, where a, p, q, and r are rational numbers in simplest form.
LEVEL 3	Rewrite expressions with several factors involving radicals and/or rational exponents.	Rewrite $\sqrt[3]{27x^5}$ as an expression of the form a^px^p, where a and p are rational numbers.
LEVEL 2	Rewrite numerical expressions containing rational exponents in simplest radical form.	Express $(27)^{\frac{1}{2}}$ in simplest radical form.
LEVEL 1	Simplify numerical radicals.	Simplify $\sqrt[3]{27}$.

QUANTITIES (N-Q)

A. Reason quantitatively and use units to solve problems.

(N-Q.A.2) Define appropriate quantities for the purpose of descriptive modeling.

Emphasis:	■■□ Supporting content (14-28% of Regents)
Notes:	Shared with Algebra I. This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude. (PARCC)
Related:	Create one-variable equations and inequalities (A-CED.A.1) Write a function to describe a relationship (F-BF.A.1) Write arithmetic and geometric sequences explicitly and recursively, translate between the forms, use for modeling (F-BF.A.2) Use sine or cosine functions to model periodic behavior (F-TF.B.5) Construct linear and exponential functions, incl. arithmetic and geometric sequences (F-LE.A.2) Use sine or cosine functions to model periodic behavior (F-LE.B.5)

ALGEBRA (35-44% OF REGENTS EXAM)

THE COMPLEX NUMBER SYSTEM (N-CN)

A. Perform arithmetic operations with complex numbers.

(N-CN.A.1) Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$, with a and b real.
(N-CN.A.2) Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract and multiply complex numbers.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: (N-CN.A.1)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Simplify expressions containing variables, and sums and products of complex numbers in $a + bi$ form.	Simplify $(x - 6i)^4$, where i is the imaginary unit.
LEVEL 4 *(meets standard)*	Simplify expressions containing variables and sums or differences, products, and powers of complex numbers in $a + bi$ form.	Simplify $xi(i - 7i)$, where i is the imaginary unit.
LEVEL 3	Simplify expressions containing sums or differences and products of complex numbers in $a + bi$ form.	Simplify $12 + i - (4 + 7i)(9 - 4i)$.
LEVEL 2	Simplify products of complex numbers in $a + bi$ form.	Simplify $(3 + 6i)(7 - 4i)$.
LEVEL 1	Simplify sums and differences of complex numbers in $a + bi$ form.	Simplify $(2 + 7i) - (8 - 12i)$.

C. Use complex numbers in polynomial identities and equations.

(N-CN.C.7) Solve quadratic equations with real coefficients that have complex solutions.

Emphasis:	■□□ Additional content (19-33% of Regents)
Related:	Identify zeroes of quadratic and cubic polynomials and use them to sketch graphs (A-APR.B.3) Solve quadratic equations by inspection, taking square roots, factoring, completing the square, quadratic formula, graphing) (A-REI.B.4) (N-CN.A.1) Sketch graphs of functions given verbal description, interpret key features of graphs and tables (F-IF.B.4) Graph and show features of graphs (F-IF.C.7) Graph polynomial functions and show zeroes and end behavior (F-IF.C.7c)

SEEING STRUCTURE IN EXPRESSIONS (A-SSE)

A. Interpret the structure of expressions.

(A-SSE.A.2) Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Includes factoring by grouping. (NYSED) Tasks are limited to polynomial, rational, or exponential expressions. Examples: $x^2 + 2x + 1 + y^2 = 9 = (x + 1)^2 + y^2 = 9$ is a circle with radius 3 and center (-1, 0), $\frac{x^2+4}{x^2+3} = \frac{(x^2+3)+1}{x^2+3} = 1 + \frac{1}{x^2+3}$. (PARCC)

Related: Write expressions in equivalent forms to reveal properties(A-SSE.B.3)
Rewrite exponential expressions (A-SSE.B.3c)
Justify steps in solving rational or radical equations (A-REI.A.1b)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Simplify products or quotients of rational expressions.	Simplify $\frac{x^2-x-6}{x^2+4x-5} \div \frac{x^2-7x+12}{x^2+x-20}$ and state the values for which the expression is undefined.
LEVEL 4 *(meets standard)*	Factor polynomial expressions using the sum or difference of cubes. Simplify rational expressions.	Factor $125x^3 - 27y^3$. Simplify $\frac{x^2+4}{x^2+3}$.
LEVEL 3	Rewrite polynomial expressions as equivalent expressions in terms of a sum or difference of cubes.	Rewrite $64x^3 - 8y^3$ as a difference of cubes.
LEVEL 2	Use appropriate arithmetic operations of polynomials to determine if two expressions are equivalent.	Simplify the product $(x+y)(x-y)$ and determine whether it is equivalent to $x^2 - y^2$.
LEVEL 1	Provide evidence that two expressions are equivalent by substituting numerical values for variables.	Provide evidence that $(x+y)(x-y)$ is equivalent to $x^2 - y^2$ by substituting $x = 3$ and $y = 4$ into each expression.

B. Write expressions in equivalent forms to solve problems.

(A-SSE.B.3) Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Emphasis:	■■■ Major content (51-65% of Regents)
Notes:	Shared with Algebra I. Tasks have a real-world context. As described in the standard, there is interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. Tasks are also limited to exponential expressions with rational or real exponents. (PARCC)
Related:	Rewrite exponential expressions (A-SSE.B.3a) Rewrite exponential expressions (A-SSE.B.3c) Write a function in different forms to reveal its properties (F-IF.C.8) Interpret exponential functions and classify as growth or decay (F-IF.C.8b)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain multiple interpretations of expressions in terms of its context.	The expressions $(1.00427)^x$ and $(1.000984)^y$ are different representations of $(1.0525)^t$, which represents a 5.25% annual increase in revenue for a company after *t* years. Explain in context what $(1.00427)^x$ and $(1.000984)^y$ could represent.
LEVEL 4 *(meets standard)*	Rewrite a polynomial, rational, exponential, or trigonometric expression to reveal properties of the quantity represented by the expression.	(Jun. 2016 Algebra II, #16) Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, write an expression that the company's chief financial officer could use to approximate their monthly percent increase in revenue. (Let *m* represent months.)
LEVEL 3	Identify the parts of a polynomial, rational, exponential, or trigonometric function in a real-world context.	The amount of money in a bank account after *t* years of investment can be modeled using the formula $f(t)=750\left(1+\frac{0.05}{12}\right)^{12t}$. Explain what the numbers 0.05 and 12 represent.
LEVEL 2	Identify the parts of a polynomial, rational, exponential, or trigonometric function.	Identify the rate of change in the function $f(x) = 100(1 + 0.05)^x$.
LEVEL 1	Determine if polynomial, rational, exponential, or trigonometric expressions are equivalent.	Is $100(1.05)^3$ equivalent to 105^3?

(A-SSE.B.3c) Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.01212^t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Emphasis: ■■■ Major content (51-65% of Regents)
Notes: Shared with Algebra I. Tasks have a real-world context. As described in the standard, there is interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. Tasks are also limited to exponential expressions with rational or real exponents. (PARCC)
Related: Write expressions in equivalent forms to reveal properties (A-SSE.B.3)

	TASK	**EXAMPLE**
LEVEL 5 *(exceeds standard)*	Transform and compare exponential functions with different compounding periods.	Allied Bank offers an investment account that has an annual percentage rate of 0.22% compounded monthly. Best Bank offers an investment account that has an annual percentage rate of 0.2% compounded weekly. If the same amount is invested in each account, which account would have a higher balance after 1 year? Justify your answer.
LEVEL 4 *(meets standard)*	Transform exponential functions to show that they are equivalent and interpret the transformations in context.	(Jun. 2016 Algebra II, #16) Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, write an expression that the company's chief financial officer could use to approximate their monthly percent increase in revenue. (Let m represent months.)
LEVEL 3	Use the properties of exponents to show that two exponential variable expressions with different bases and integer exponents are equivalent.	Is $(8)2^t$ equivalent to 2^{t+3}? Explain.
LEVEL 2	Use the properties of exponents to determine if two exponential variable expressions with the same base and integer exponents are equivalent.	Is $(2^{3a})(2^b)$ equivalent to 2^{3a+b}? Explain.
LEVEL 1	Determine if two exponential expressions without variables are equivalent.	(Jan. 2010 Int. Alg. Regents, #20) Which expression is equivalent to $3^3 \bullet 3^4$? (1) 9^{12} (2) 9^7 (3) 3^{12} (4) 3^7

(A-SSE.B.4) Derive the formula for the sum of a finite geometric series and use the formula to solve problems. For example, calculate mortgage payments.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Includes using summation notation (NYSED).

Related: Construct linear and exponential functions, incl. arithmetic and geometric sequences (F-LE.A.2)
Identify explicit and recursive sequences as functions with integer domain (F-IF.A.3)
Write arithmetic and geometric sequences explicitly and recursively, translate between the forms, use for modeling (F-BF.A.2)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Prove identities that use summation notation. Prove the formulas for finite geometric series. Use summation notation to represent series that are not arithmetic or geometric.	Prove $\sum_{i=1}^{n} kx_i = k\sum_{i=1}^{n} x_i$, where *k* is a constant. Represent $x+\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}+\ldots+\sqrt[10]{x}$ using summation notation.
LEVEL 4 *(meets standard)*	Represent the sum of a finite geometric series using summation notation. Evaluate the sum of a finite geometric series written in summation notation. Apply the geometric series formula to solve a real world problem.	Represent 1 + 3 + 9 + 27 + … + 531,441 using summation notation. Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, S_n, for Alexa's total earnings over *n* years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent.*
LEVEL 3	Evaluate a finite geometric series by substituting given values into the formula for a geometric series. Express a geometric series in the form $a_1 + a_1r + a_1r^2 + \ldots + a_1r^{n-1}$.	Evaluate 1 + 3 + 9 + 27 + … + 531,441. Express the series 3 + 8 + 13 + 18 + … + 143 in the form $a_1 + a_1d + a_1(2d) + \ldots + a_1(n-1)d$.
LEVEL 2	Determine whether a series is geometric.	Is the series 1 + 3 + 5 + 7 + … + 99 geometric?
LEVEL 1	Define a geometric series.	What is a geometric series?

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS (A-APR)

A. Understand the relationship between zeros and factors of polynomials.

(A-APR.B.2) Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$. (PARCC)

Related: Identify zeroes of quadratic and cubic polynomials and use them to sketch graphs (A-APR.B.3)
Divide polynomials with remainder, incl. with long division (A-APR.D.6)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine the remainder when a polynomial $p(x)$ is divided by a linear expression $ax + b$, where a and b are numbers and $a \neq 1$. Use the Remainder Theorem to express in standard form the polynomial function that is defined by several given input-output pairs.	Find the remainder for the division $(32x^{40} + 2x + 1) \div (2x - 1)$. Use the Remainder Theorem to write a polynomial function $p(x)$ in standard form such that $p(1) = 5$, $p(2) = 11$, and $p(3) = 25$.
LEVEL 4 *(meets standard)*	Given a zero of a polynomial function, use the Remainder Theorem to determine the other zeros. Find the value of a coefficient of a polynomial that produces a given remainder when the polynomial is divided by a given linear expression.	(Fall 2015 Algebra II Regents Sampler, #15) Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$. Find the value of k so that $(x^3 - kx^2 + 2) \div (x - 1)$ has remainder 8.
LEVEL 3	Use the Remainder Theorem to determine the remainder when a polynomial $p(x)$ is divided by $x - a$, where a is a number.	Find the remainder for the division $(x^{40} + 3x^3 + 144) \div (x - 1)$.
LEVEL 2	Use the Remainder Theorem to determine if $x - a$, where a is a number, is a factor of a polynomial $p(x)$.	Is $x + 1$ a factor of $2x^{50} - 4x^4 + 9x^3 - x + 13$? Justify your answer.
LEVEL 1	Evaluate a polynomial function for a real value.	If $P(x) = x^3 - 5x^2 - 4$, find $P(-4)$.

B. Understand the relationship between zeros and factors of polynomials.

(A-APR.B.3) Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Emphasis: ■■□ Supporting content (14-28% of Regents)

Notes: Shared with Algebra I. Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$. (PARCC)

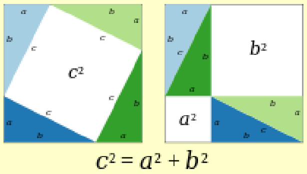
Related: Write expressions in equivalent forms to reveal properties (A-SSE.B.3)
Solve quadratic equations by inspection, taking square roots, factoring, completing the square, quadratic formula, graphing) (A-REI.B.4)
Write a function in different forms to reveal its properties (F-IF.C.8)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Identify zeros of quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided, and use the factors to graph the function in context.	Part of the design for a company logo for an outdoor sports gear company contains a green curve that intersects a horizontal brown line. The curve is generated using the cubic function $f(x) = x^3 - 33x^2 + 272x - 240$ and the horizontal line used is $y = 0$. The company logo will be printed on a large banner using the scale 1 unit = 1 foot. If the curve intersects the brown line at $x = 1$, sketch a graph of $f(x)$.
LEVEL 4 *(meets standard)*	Identify zeros of quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided, and use the factors to graph the function.	If $p(x) = x^3 - 2x^2 - x + 2$, find the zeros of $p(x)$ and use the zeros to graph the function.
LEVEL 3	Identify zeros of quadratic, cubic, and quartic polynomials given in factored form and use the factors to graph the function.	If $p(x) = (x - 5)(x - 9)(x - 12)$, find the zeros of $p(x)$ and use the zeros to graph the function.
LEVEL 2	Identify zeros of quadratic, cubic, and quartic polynomials not written in factored form.	If $p(x) = 4x^3 + 2x^2 - 36x - 18$, find the zeros of $p(x)$.
LEVEL 1	Identify the zeros of a polynomial function given in factored form.	If $p(x) = (x + 4)(x + 12)(x - 8)$, find the zeros of $p(x)$.

C. Use polynomial identities to solve problems.

(A-APR.C.4) Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Justify steps in solving rational or radical equations (A-REI.A.1)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain how a geometric illustration proves a polynomial identity.	Explain how the diagram below can be used to the Pythagorean Theorem.  (Image by William B. Faulk - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=36886381 .)
LEVEL 4 *(meets standard)*	Prove that a polynomial equation is an identity and use the identity to describe numerical relationships.	Prove that $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ is an identity and show that it can be used to generate Pythagorean triples.
LEVEL 3	Prove that a polynomial equation is an identity.	Prove that $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ is an identity.
LEVEL 2	Provide justification for a step of a given identity proof.	In her proof of the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$, Maria wrote the following for the first step: $(x^2 + y^2)^2 - (x^2 + y^2)^2 = [(x^2 + y^2) + (x^2 + y^2)][(x^2 + y^2)^2 - (x^2 + y^2)^2]$ What justification can she provide for this?
LEVEL 1	Provide evidence that an equation is an identity by substituting numerical values for the variables.	Substitute $x = 1$ and $y = 2$ into the equation $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ and determine if the result a true equation.

D. Rewrite rational expressions.

(A-APR.D.6) Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Emphasis:	■■☐ Supporting content (14-28% of Regents)
Related:	Rewrite expressions in equivalent forms (A-SSE.A.2) Write expressions in equivalent forms to reveal properties(A-SSE.B.3) Identify zeroes of quadratic and cubic polynomials and use them to sketch graphs (A-APR.B.3)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Divide polynomials and use the results to provide evidence for mathematical statements about *n*th-degree polynomials.	In the examples below, you will find several quotients and find a pattern. a. Express $(x^2 - 9) \div (x + 3)$ in standard form. b. Express $(x^3 - 27) \div (x + 3)$ in standard form. c. Express $(x^4 - 81) \div (x + 3)$ in standard form. d. Express $(x^5 - 27) \div (x + 3)$ in standard form. e. For which positive integers n is $x + 3$ a factor of $x^n - 3^n$? Explain your reasoning.
LEVEL 4 *(meets standard)*	Divide polynomials with zero coefficients to get a polynomial with lesser degree and a remainder.	Simplify $\frac{2x^3 - 4x^2 + 2}{2x - 2}$.
LEVEL 3	Divide polynomials with nonzero coefficients to get a polynomial with lesser degree and no remainder.	Simplify $(4x^3 + 19x^2 + 26x + 15) \div (x + 3)$.
LEVEL 2	Divide polynomials by factoring.	Simplify $(x^2 + 9x + 20) \div (x + 5)$.
LEVEL 1	Determine if a rational expression is equivalent to a given factored expression with no remainder.	Is $x^2 + 8x + 15$ equivalent to $(x + 3)(x + 5)$? Show appropriate calculations to justify your answer.

CREATING EQUATIONS (A-CED)

A. Create equations that describe numbers or relationships.

(A-CED.A.1) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Emphasis: ■■□ Supporting content (14-28% of Regents)

Notes: Shared with Algebra I. Tasks are limited to exponential equations with rational or real exponents and rational functions. Tasks have a real-world context. (PARCC)

Related: Solve quadratic equations by inspection, taking square roots, factoring, completing the square, quadratic formula, graphing) (A-REI.B.4).

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain how a created equation or inequality models a context.	The equation $v = 32{,}000(0.81)^{\frac{t}{12}}$ represents the value of a car after t months of ownership. Explain in context what each quantity in the equation represents.
LEVEL 4 *(meets standard)*	Create an equation or inequality in one variable and use it to solve a problem.	A colony of bacteria grew from a population of 15,000 to 250,000 in six hours. Assuming that the growth rate is exponential, approximate the growth rate to the nearest tenth of a percent.
LEVEL 3	Create an equation or inequality in one variable to represent a problem.	Alice can paint a fence in 4 hours by herself. Ben can paint the same fence in 5 hours by himself. Write an expression that represents the rate that Alice paints, the rate that Ben paints, and the rate that both would paint if working together. Let x represent the number of hours that Alice and Ben would spend if they painted the fence together.
LEVEL 2	Identify an unknown quantity from a context.	Alice can paint a fence in 4 hours by herself. Ben can paint the same fence in 5 hours by himself. The equation $\frac{1}{4}+\frac{1}{5}=\frac{1}{x}$ can be used to represent this situation. Explain what $\frac{1}{x}$ means in this context.
LEVEL 1	Solve an equation or inequality in one variable.	To the nearest tenth, solve the equation $200 = 50(3^x)$ for x.

REASONING WITH EQUATIONS AND INEQUALITIES (A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

(A-REI.A.1) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Emphasis: ■■■ Major content (51-65% of Regents)
Notes: Shared with Algebra I. Tasks are limited to simple rational or radical equations. (PARCC)
Related: Prove and use polynomial identities (A-APR.C.4)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine the best method of solving a radical or rational equation. Explain why extraneous roots arise while solving radical or rational equations.	Explain why squaring both sides of the equation $\sqrt{x+5}-4=\sqrt{x+1}$ is inefficient. Determine which method of solving the equation $\frac{x-1}{2}+\frac{3x+1}{5x}=\frac{7}{3}$ is better – multiplying both sides of the equation by the least common denominator or finding a common denominator for all fractions. Justify your answer.
LEVEL 4 *(meets standard)*	Justify all steps in the solution of a radical or rational equation using appropriate properties of equality.	Solve the equation $\frac{x-1}{2}+\frac{3x+1}{5x}=\frac{7}{3}$ and justify each step using an appropriate property of equality.
LEVEL 3	Justify a step in the solution of a radical or rational equation with an appropriate property of equality.	When solving the equation $\sqrt{x+5}-4=\sqrt{x+1}$, Emily wrote $x+5-8\sqrt{x+5}+16=x+1$ as an intermediate step. What property of equality justifies this work?
LEVEL 2	Write radical or rational equations that illustrate a given property of equality.	Write two radical equations that illustrate the addition property of equality.
LEVEL 1	State a property of equality.	State the addition property of equality.

(A-REI.A.2) Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Emphasis: ■■■ Major content (51-65% of Regents)

Related: Solve quadratic equations by inspection, taking square roots, factoring, completing the square, quadratic formula, graphing) (A-REI.B.4).

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Solve rational equations whose numerators' polynomial degree is greater than 1 and identify extraneous solutions. Solve equations with two radical expressions and identify extraneous solutions.	Solve for m in the equation $\frac{m+5}{m^2+m}=\frac{1}{m^2+m}-\frac{m-6}{m+1}$. Solve for x in the equation $\sqrt{2x-8}+\sqrt{3x-12}=0$.
LEVEL 4 *(meets standard)*	Solve rational equations with linear numerators and identify extraneous solutions. Solve radical equations in one variable (with one radical expression) and identify extraneous solutions.	Solve for the variable: $\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b-2}{b^2-9}$. Solve for x in the equation $\sqrt{x^2-8}-x=4$.
LEVEL 3	Solve more complicated rational equations (with linear numerators) or radical equations (with one radical expression) in one variable that do not have extraneous solutions.	Solve for the variable: $\frac{2x+7}{6}-\frac{2x-9}{10}=3$. Solve for x in the equation $\sqrt{2x+5}=7$.
LEVEL 2	Solve rational equations (with monomial linear numerators) or radical equations (with one monomial radical expression) in one variable that do not have extraneous solutions.	Solve for the variable: $\frac{2x}{5}+\frac{3x}{10}=7$. Solve for the variable: $\sqrt{4x}=2$.
LEVEL 1	Verify that a number is a solution to a rational or radical equation.	Is 4 a solution to the equation $\frac{x-4}{17}=\frac{5x}{2}$? Is -36 a solution to the equation $\sqrt{72-x}=6$?

B. Solve equations and inequalities in one variable.

(A-REI.B.4) Solve quadratic equations in one variable.

(A-REI.B.4b) Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Emphasis:	■■□ Supporting content (14-28% of Regents)
Notes:	Shared with Algebra I. In the case of equations that have roots with nonzero imaginary parts, students write the solutions as $a \pm bi$ for real numbers a and b.
Related:	Solve rational and radical equations, identify extraneous solutions (A-REI.A.2) Solve a quadratic-linear system of equations algebraically and graphically (A-REI.C.7) (N-CN.A.1) Write a function in different forms to reveal its properties (F-IF.C.8)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine the nature of the roots of a quadratic equation without solving. Determine the sum and product of the roots of a quadratic equation.	Determine if the roots of the equation $14x^2 + 56x - 3 = 0$ are rational, irrational, or imaginary. Are the roots equal or unequal? Find the sum of the roots of the equation $8x^2 + 16x - 5 = 0$.
LEVEL 4 *(meets standard)*	Solve quadratic equations in one variable with complex solutions and write the solutions in $a + bi$ form.	Solve for the variable and express solutions in simplest $a + bi$ form: $4x^2 - 7x - 3 = 0$.
LEVEL 3	Factor quadratic expressions in one variable. Given a quadratic expression, identify an equivalent expression in completed-square form.	Write an expression equivalent to $x^2 + 4x + 5$ in completed-square form.
LEVEL 2	Solve quadratic equations with complex numbers in the form $x^2 = a$.	Solve $x^2 = -49$ for x.
LEVEL 1	Verify that a complex number is a solution to a quadratic equation.	Determine if $x = 4 + i$ is a solution to $x^2 - 8x + 17 = 0$.

C. Solve systems of equations.

(A-REI.C.6) Solve systems of three equations in three variables.

Emphasis: ■□□ Additional content (19-33% of Regents)
Notes: Exclusively tested on three equations in three unknowns. (NYSED)
Related: Solve a quadratic-linear system of equations algebraically and graphically (A-REI.C.7)
Approximate, justify, interpret graphical solution to $f(x) = g(x$ (A-REI.D.11)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Write the equation of a parabola that passes through three given points.	A parabola passes through the points (1, 6), (3, 20), and (-2, 15). Substitute the ordered pairs into the standard form of a parabola $y = ax^2 + bx + c$ to write three linear equations in terms of a, b, and c. Then use those equations to write an equation of the parabola.
LEVEL 4 *(meets standard)*	Solve systems of three equations in three variables algebraically.	Solve for the variables: $2a + 4b + c = 5$ $a - 4b = -6$ $2b + c = 7$
LEVEL 3	Solve systems of two equations in two variables using addition and multiplication.	Solve for the variables: $4x + 2y = 14$ $7x - 3y = -8$
LEVEL 2	Solve systems of two equations in two variables using substitution or addition.	Solve for the variables: $x + y = 22$ $x - y = 10$
LEVEL 1	Verify a solution to a system of two equations in two variables.	Verify that (2, 6) is a solution to the system: $4x + 2y = 14$ $7x - 3y = -8$

(A-REI.C.7) Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Approximate, justify, interpret graphical solution to $f(x) = g(x$ (A-REI.D.11).

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Solve a system of three linear equations in three variables algebraically. Solve a system of two quadratic equations. Solve a system consisting of a linear equation and a quadratic equation (representing an ellipse).	Solve for the variables: $r = 2(s - t)$ $2t = 3(s - r)$ $r + t = 2s - 3$ Solve for the variables: $\frac{x^2}{2} + \frac{(y-2)^2}{9} = 22$ $5x - y = 34$
LEVEL 4 *(meets standard)*	Solve a system consisting of a linear and a quadratic equation in two variables.	Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
LEVEL 3	Solve a system of two linear equations in two variables.	Solve for the variables: $4x + 2y = 14$ $7x - 3y = -8$
LEVEL 2	Solve a linear or quadratic equation in one variable.	Solve for the variable: $4x + 12 = 14$.
LEVEL 1	Graph a linear or quadratic equation in two variables and state the coordinates of points on the graph.	Graph the equation $y = x^2 + 12x + 32$. State the coordinates of three points on the graph.

D. Represent and solve equations and inequalities graphically.

(A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Emphasis:	■■■ Major content (51-65% of Regents)
Notes:	Shared with Algebra I.
Related:	Solve a quadratic-linear system of equations algebraically and graphically (A-REI.C.7) Derive equation of parabola given focus and directrix Derive equation of parabola given focus and directrix (G-GPE.A.2)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Find the solution of $f(x) < g(x)$ or $f(x) \le g(x)$ algebraically.	Solve the inequality $x^2 - x - 6 \le 2x^2 + 4x$ algebraically.
LEVEL 4 *(meets standard)*	Approximate the solutions to $f(x) = g(x)$. Interpret the solutions to $f(x) = g(x)$ in context.	(January 2017 Algebra II Regents, #16) Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?
LEVEL 3	Graph a system of two functions and determine the number of points of intersection.	Graph the functions $f(x) = 4\sin 2x$ and $g(x) = 3\cos\frac{x}{4}$ on the coordinate plane and determine the number of points of intersection.
LEVEL 2	Determine if a value is a solution to $f(x) = g(x)$.	Determine if the value $x = 3$ is a solution to the equation $2^x + 1 = \log x + 3$.
LEVEL 1	Make a table of values for a given function.	Make a table for values for the function $f(x) = x^2 + 6x + 8$ for $x = \{-5, -4, -3, -2, -1\}$.

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (G-GPE)

A. Translate between the geometric description and the equation for a conic section

(G-GPE.A.2) Derive the equation of a parabola given a focus and directrix.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Approximate, justify, interpret graphical solution to $f(x) = g(x$(A-REI.D.11)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Derive the formula for a parabola with vertex (h, k) and distance from the focus to directrix of $p > 0$.	Derive the formula for a parabola with vertex (h, k) and distance from the focus to directrix of $p > 0$.
LEVEL 4 *(meets standard)*	Write the equation of a parabola given its focus and directrix.	Write an equation of the parabola that has a focus of (1, 3) and a directrix of $y = 1$.
LEVEL 3	Write expressions that represent the distance between a point (x, y) on a parabola and its focus and directrix.	A parabola that has a focus $F(0, 4)$ and a directrix d (whose equation is $y = 2$) passes through the point $P(x, y)$. Write expressions that represent the distance PF and the distance from P to d.
LEVEL 2	Verify that a given point on a parabola is equidistant from its focus and directrix.	A parabola that has a focus of (0, 4) and a directrix of $y = 2$ passes through the point (4, 7). Verify that the point (4, 7) is equidistant from the parabola's focus and directrix.
LEVEL 1	State the definition of a parabola, including the terms focus and directrix. Find the distance between two points.	Find the distance between the points (4, –2) and (0, 4) in simplest radical form.

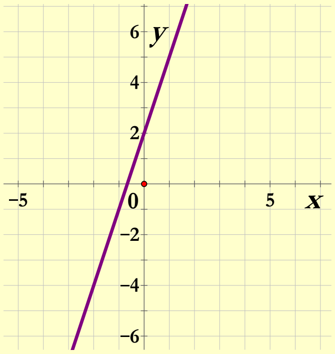
FUNCTIONS (30%-40% OF REGENTS EXAM)

INTERPRETING FUNCTIONS (F-IF)

A. Understand the concept of a function and use function notation

(F-IF.A.3) Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.

Emphasis: ■■□ Supporting content (14-28% of Regents)
Notes: Shared with Algebra I. This standard should support the major work in F-BF.2 for coherence.
Related: Construct linear and exponential functions, incl. arithmetic and geometric sequences (F-LE.A.2)
Write arithmetic and geometric sequences explicitly and recursively, translate between the forms, use for modeling (F-BF.A.2)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Differentiate between sequences and corresponding functions whose domains are real numbers.	Explain why the sequence {…, –4, –1, 2, 5, …} cannot be represented by the graph below: 
LEVEL 4 *(meets standard)*	Given a sequence written as a function rule, evaluate the function for a given input. Given two terms in a sequence, find a formula for the *n*th term.	If $f(1) = 3$ and $f(n) = -2f(n - 1) + 1$, then $f(5) =$ (1) –5 (2) 11 (3) 21 (4) 43
LEVEL 3	Identify an explicitly or recursively defined sequence as a function.	A sequence is defined recursively as follows: the first term is 4 and each subsequent term is 3 more than twice the previous term. Is this a function? Explain.
LEVEL 2	State the definition of a function.	State the definition of a function.
LEVEL 1	Identify and continue patterns of arithmetic or geometric sequences.	Write the next three numbers that follow the pattern of the sequence 3, 6, 12, 24, ….

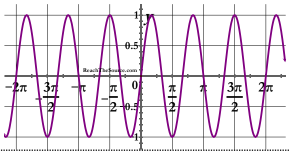
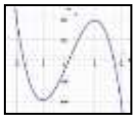
B. Interpret functions that arise in applications in terms of the context

(F-IF.B.4) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Shared with Algebra I. Tasks have a real-world context. Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. (PARCC)

Related: Graph and show features of graphs Graph and show features of graphs(F-IF.C.7)
Graph polynomial functions and show zeroes and end behavior (F-IF.C.7c)
(F-IF.C.7e)
Write a function in different forms to reveal its properties (F-IF.C.8)
Interpret exponential functions and classify as growth or decay (F-IF.C.8b)
Compare properties of two functions represented in different ways (F-IF.C.9)
Interpret parameters of linear or exponential function in context (F-LE.B.5)

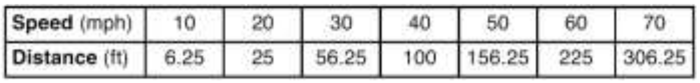
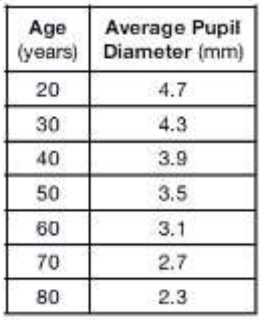
	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Given a verbal description of the relationship between two quantities, sketch a graph of the step function that models it. Given a step function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities.	New York City taxicab fares consist of an initial charge of \$2.50 and an additional charge of \$0.50 per 1/5 mile. Sketch a graph of New York City taxicab fares as a function of distance in miles.
LEVEL 4 *(meets standard)*	Given a verbal description of the relationship between two quantities, sketch a graph of the function that models it. Given a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities.	(Aug. 2016 Alg. II, #25) The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.
LEVEL 3	Given a graph of a function, determine its key features.	Determine the amplitude, period, midline, and frequency of the graph of the function shown below: 
LEVEL 2	Given a verbal description of the relationship between two quantities, determine the relationship's type.	A passenger car on a Ferris wheel makes one complete rotation around the wheel every 5 minutes. What type of function best approximates the height of the wheel above the ground over a 20-minute period – polynomial, logarithmic, or trigonometric?
LEVEL 1	Given a graph of a function, determine its type.	Determine whether the graph below represents a polynomial function. If so, determine the lowest possible degree of the polynomial. 

(F-IF.B.6) Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Shared with Algebra I. Tasks have a real-world context. Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. (PARCC)

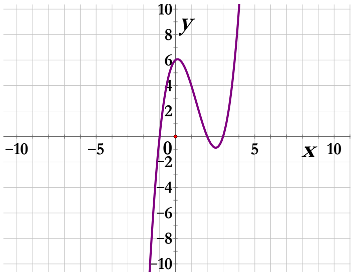
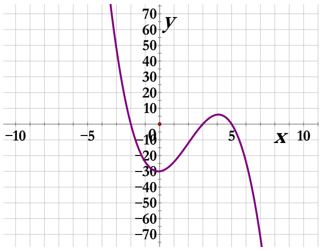
<table>
<tr><th></th><th>TASK</th><th>EXAMPLE</th></tr>
<tr><td>LEVEL 5
<i>(exceeds standard)</i></td><td>Generate a function that illustrates a given rate of change.</td><td>Give an example of a polynomial function whose average rate of change over successive intervals of 10 units increases as x increases from 5 to $+\infty$ and decreases as x decreases from 5 to $-\infty$.</td></tr>
<tr><td>LEVEL 4
<i>(meets standard)</i></td><td>Calculate, interpret, and compare the relationship between the average rates of change of two polynomial, exponential, trigonometric or logarithmic functions over a specified interval.
Calculate, interpret, and compare the relationship between the average rates of change of a polynomial, exponential, trigonometric or logarithmic functions over specified intervals.</td><td>(Jan. 2017 Alg. II Regents, #21) Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of B dollars after m months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after m months. Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

<table>
<tr><th>m</th><th>B</th></tr>
<tr><td>0</td><td>1000.00</td></tr>
<tr><td>10</td><td>1172.00</td></tr>
<tr><td>19</td><td>1352.00</td></tr>
<tr><td>36</td><td>1770.80</td></tr>
<tr><td>60</td><td>2591.90</td></tr>
<tr><td>69</td><td>2990.00</td></tr>
<tr><td>72</td><td>3135.80</td></tr>
<tr><td>73</td><td>3186.00</td></tr>
</table></td></tr>
<tr><td>LEVEL 3</td><td>Calculate and interpret the average rate of change for a function over a specified interval given a table of values.</td><td>(Jan. 2016 Alg. II Regents, #31) The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds. Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

<table>
<tr><td>Speed (mph)</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td></tr>
<tr><td>Distance (ft)</td><td>6.25</td><td>25</td><td>56.25</td><td>100</td><td>156.25</td><td>225</td><td>306.25</td></tr>
</table></td></tr>
<tr><td>LEVEL 2</td><td>Calculate and interpret the rate of change for a linear function over a specified interval.</td><td>(Aug. 2014 Alg. I Regents, #14) The table below shows the average diameter of a pupil in a person's eye as he or she grows older. What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?
(1) 2.4
(2) 0.04
(3) −2.4
(4) −0.04

<table>
<tr><th>Age (years)</th><th>Average Pupil Diameter (mm)</th></tr>
<tr><td>20</td><td>4.7</td></tr>
<tr><td>30</td><td>4.3</td></tr>
<tr><td>40</td><td>3.9</td></tr>
<tr><td>50</td><td>3.5</td></tr>
<tr><td>60</td><td>3.1</td></tr>
<tr><td>70</td><td>2.7</td></tr>
<tr><td>80</td><td>2.3</td></tr>
</table></td></tr>
<tr><td>LEVEL 1</td><td>Identify the rate of change given the equation of a linear function.
Distinguish between graphs of increasing and decreasing functions.</td><td>If $f(x) = -3x + 6$, what is the rate of change of the function?</td></tr>
</table>

C. Analyze functions using different representations

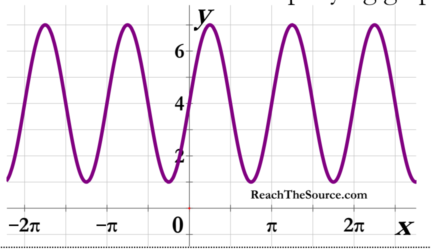
(F-IF.C.7) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
(F-IF.C.7c) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Emphasis: ■■□ Supporting content (14-28% of Regents)
Notes: Shared with Algebra I.
Related: Write expressions in equivalent forms to reveal properties (A-SSE.B.3)
Identify zeroes of quadratic and cubic polynomials and use them to sketch graphs (A-APR.B.3)
Sketch graphs of functions given verbal description, interpret key features of graphs and tables (F-IF.B.4)
Graph polynomial functions and show zeroes and end behavior (F-IF.C.7c)
(F-IF.C.7e)
Write a function in different forms to reveal its properties (F-IF.C.8)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Given a verbal description of the key features of the graph of a function, sketch the graph accurately and express it symbolically.	Graph and write an equation in standard form for the lowest-degree polynomial function whose zeroes are 1, -1, and 5 and whose leading coefficient is 1.
LEVEL 4 *(meets standard)*	Graph a polynomial function of degree 3 or higher expressed symbolically and identify its intercepts, maxima, and minima from the graph.	Graph $f(x) = (x + 3)(x + 6)(x - 1)$ on the coordinate plane and identify its intercepts, maxima, and minima from the graph.
LEVEL 3	Identify the intercepts, maxima, and minima from the equation of a function.	Identify the intercepts, and relative maxima, and minima of the function $f(x) = (x + 3)(x + 6)(x - 1)$.
LEVEL 2	Identify the intercepts, maxima, and minima from a graph of a function.	Identify the intercepts, maxima, minima, and end behavior of the polynomial function whose graph appears below. 
LEVEL 1	Identify the degree of a polynomial function given its graph.	State the degree of the polynomial function whose graph is shown below. 

(F-IF.C.7e) Graph exponential and log functions, showing intercepts and end behavior, and trig functions, showing period, midline and amplitude.

Emphasis: ■■□ Supporting content (14-28% of Regents)
Notes: Shared with Algebra I.
Related: Write expressions in equivalent forms to reveal properties (A-SSE.B.3)
Identify zeroes of quadratic and cubic polynomials and use them to sketch graphs (A-APR.B.3)
Sketch graphs of functions given verbal description, interpret key features of graphs and tables (F-IF.B.4)
Graph and show features of graphs (F-IF.C.7)
Graph polynomial functions and show zeroes and end behavior (F-IF.C.7c)
(F-IF.C.7e)
Write a function in different forms to reveal its properties (F-IF.C.8)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	State the relationship between the key features of the graphs of an exponential, logarithmic, or trigonometric function and a transformation of the function.	Explain how the end behavior and intercepts of $y = b^x$ compare to the end behavior and intercepts of $y = ab^{x+m} + n$.
LEVEL 4 *(meets standard)*	Graph an exponential or logarithmic function with given intercepts and end behavior. Graph a trigonometric function with given period, midline, and amplitude.	(Jun. 2016 Alg. II, #28) Graph *one* cycle of a cosine function with amplitude 3, period $\pi/2$, midline $y = -1$, and passing through the point (0, 2).
LEVEL 3	Given a graph of an exponential, logarithmic, or trigonometric function, state its key features.	State the period, midline, and amplitude of the trigonometric function shown in the accompanying graph. 
LEVEL 2	Graph $y = \cos x$ or $y = \sin x$ and state its period, midline, and amplitude. State the relationship between the midline and amplitude of a sine or cosine function.	Graph $y = \cos x$ and state its period, midline, and amplitude.
LEVEL 1	Graph $y = b^x$ or $y = \log_b x$, where b is a whole number, and state its intercepts and end behavior.	Graph and state the intercepts and end behavior of $y = 2^x$.

(F-IF.C.8) Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

(F-IF.C.8b) Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Emphasis: ■■□ Supporting content (14-28% of Regents)

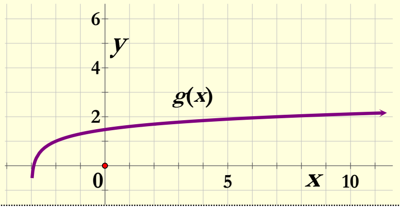
Notes: Shared with Algebra I. Includes $A = Pe^{rt}$ and $A = P\left(1+\frac{r}{n}\right)^{nt}$.

Related: Rewrite expressions in equivalent forms (A-SSE.A.2)
Write expressions in equivalent forms to reveal properties (A-SSE.B.3)
Compare properties of two functions represented in different ways (F-IF.C.9)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine the values of a constant that will make an exponential function represent growth or decay.	For what values of a will the function $f(x)=\left(\frac{1}{a}+1\right)^x$ represent exponential growth?
LEVEL 4 *(meets standard)*	Rewrite a function in an equivalent form to interpret its properties. Given an exponential function equation, use the properties of exponents to determine whether the function represents exponential growth or decay.	(Jun. 2016 Algebra II Regents, #16) Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, write an expression that the company's chief financial officer could use to approximate their monthly percent increase in revenue. (Let m represent months.) Determine algebraically whether the function $f(x)=\left(\frac{1}{2}\right)^{3-4x}$ represents growth or decay.
LEVEL 3	Identify the key features of a function given its equation or graph. Rewrite a function in an equivalent form.	In a study by the state department of environmental protection, he population of birds in a state park can be modeled using the function $f(x) = 65(1.14)^x$, where x is the number of years since the study began and $f(x)$ is the bird population in thousands. State the population's annual rate of growth. Rewrite the expression $(1.16)^{\frac{x}{2}}$ in the form a^x.
LEVEL 2	Identify different but equivalent forms of the same expression.	Determine if the expressions $(1.16)^{\frac{x}{2}}$ and $(1.08)^x$ are equivalent.
LEVEL 1	Define the intercepts, relative extrema, asymptotes, and other key features of the graph of a function.	What is an asymptote of a function?

(F-IF.C.9) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Emphasis: ■■□ Supporting content (14-28% of Regents)
Notes: Shared with Algebra I. Tasks may involve polynomial, exponential, logarithmic and trigonometric functions. (PARCC)
Related: Write a function in different forms to reveal its properties (F-IF.C.8)
Interpret exponential functions and classify as growth or decay (F-IF.C.8b)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain how a representation of a function shows a property of a function.	Explain how a table of values can be used to find an asymptote of the graph of a function.
LEVEL 4 *(meets standard)*	Compare the properties of two functions represented in different ways.	Given the two functions $f(x)$ and $g(x)$ below, determine which has the larger x-intercept. $f(x) = \log_3 (x - 2)$ 
LEVEL 3	Compare the properties of two functions represented in the same way.	Which function has a greater maximum: $f(x) = 2\sin 3x$ or $g(x) = \sin 4x + 2$?
LEVEL 2	State the properties of a given function represented algebraically, graphically, in tables, or by verbal descriptions.	What is the asymptote of the function $f(x) = \log_6 x + 7$?
LEVEL 1	Define the intercepts, relative extrema, asymptotes, and other key features of the graph of a function.	What is an asymptote of a function?

BUILDING FUNCTIONS (F-BF)

A. Build a function that models a relationship between two quantities

(F-BF.A.1) Write a function that describes a relationship between two quantities.

(F-BF.A.1a) Determine an explicit expression, a recursive process, or steps for calculation from a context.

Emphasis: ■■■ Major content (51-65% of Regents)

Notes: Shared with Algebra I. Tasks have a real-world context. Tasks may involve linear, quadratic, and exponential functions. (PARCC)

Related: Write a function in different forms to reveal its properties (F-IF.C.8)
Combine functions using arithmetic operations Combine functions using arithmetic operations Combine functions using arithmetic operations Combine functions using arithmetic operations Combine functions using arithmetic operations Combine functions using arithmetic operations (F-BF.A.1b)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine a recursive representation for a function.	Write a recursive representation for the function $f(x) = 2x + 3$ for $x \geq 0$.
LEVEL 4 *(meets standard)*	Determine and write the appropriate function that describes a relationship between two quantities.	Allie has $100 that she wants to put into an investment account for 5 years. The account earns 12% interest per year, compounded semi-annually. Write a function for the amount of money she would have in her account at the end of t years.
LEVEL 3	Write a qualitative or narrative description of a function that describes the behavior and/or relationship between two quantities.	The cost in dollars of mailing a letter weighing z ounces, where z is an integer greater than 1, is determined by the function $c(z) = 0.20(z - 1) + 0.46$. Explain in words how the cost of mailing the letter is determined.
LEVEL 2	Determine intermediate steps or calculations for a given function.	For the function $f(x) = 625(0.20^x) + 1.5$, explain how the output is calculated for a given input x.
LEVEL 1	Given a verbal description of a relationship between two quantities, create a table of input and output values.	In 2013, the United States Postal Service charged $0.46 to mail a letter weighing up to 1 oz. and $0.20 per ounce for each additional ounce. Create a table showing the costs of mailing letters that weigh 1, 2, 3, and 4 ounces.

(F-BF.A.1b) Combine standard function types using arithmetic operations. (For example build a function the models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate the functions to the model).

Emphasis:	■■■ Major content (51-65% of Regents)
Notes:	Shared with Algebra I. Tasks have a real-world context. (PARCC)
Related:	Write a function in different forms to reveal its properties (F-IF.C.8) Write a function to describe a relationship (F-BF.A.1)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Combine and interpret functions using arithmetic operations in context. Compose functions.	If $f(x) = 2x^2 + 1$ and $g(x) = 3(6^x) + 1$, find $f(g(x))$.
LEVEL 4 *(meets standard)*	Combine functions using multiple arithmetic operations.	If $p(x) = ab^x$, $q(x) = 5m^x$, and $r(x) = cd^x$, then find $p(x) \bullet r(x) + q(x)$.
LEVEL 3	Combine functions using an arithmetic operation.	(Jan. 2017 Algebra II Regents, #10) If $p(x) = ab^x$ and $r(x) = cd^x$, then find $p(x) \bullet r(x)$.
LEVEL 2	Combine polynomial expressions using multiple arithmetic operations.	Simplify $(2x^2)(5x^3) + 3x^5 - 7x^4$.
LEVEL 1	Add, subtract, multiply, or divide expressions.	Simplify $(ab^x)(cd^x)$.

(F-BF.A.2) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situation, and translate between the two forms.

Emphasis: ■■■ Major content (51-65% of Regents)

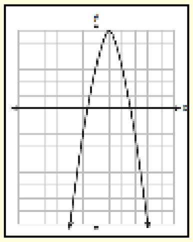
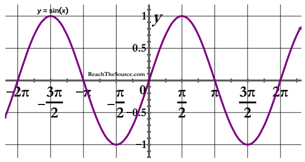
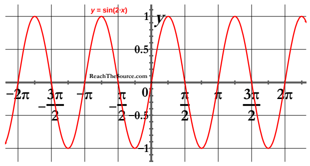
Related: Identify explicit and recursive sequences as functions with integer domain (F-IF.A.3)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Justify that a specific recursively-defined and explicit formula represent the same sequence. Given two terms of a sequence, find another term.	The second term of a geometric sequence is 12 and the fifth term is –96. Find the ninth term of the sequence.
LEVEL 4 *(meets standard)*	Determine and write the function that generates an arithmetic or geometric sequence in a real-world context.	(August 2016 Algebra II Regents, #24) In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, write an equation that can be used to predict the population of New York State t years after 2010.
LEVEL 3	Translate between the explicit and recursive forms of a sequence.	Write an explicit formula for the sequence $a_1 = 120$, $a_n = 0.8a_{n-1}$ for $n \geq 2$.
LEVEL 2	Given several terms of a sequence, write its explicit or recursive formula. Determine if a given sequence is arithmetic, geometric, or neither.	Write a recursive formula for the sequence 6, 24, 96, 384, …. Determine if the sequence 1, 1, 2, 3, 5, 8, 13, … is arithmetic, geometric, or neither.
LEVEL 1	Write terms for an arithmetic or geometric sequence given its formula. Determine if a given sequence is represented recursively or explicitly.	Write the first five terms of the sequence $a_n = 4(3^n)$. Is the sequence represented by the formula $a_1 = -2$ and $a_n = (-3)a_{n-1} + 4$ for $n \geq 2$ recursive or explicit?

B. Build new functions from existing functions

(F-BF.B.3) Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Emphasis: ■□□ Additional content (19-33% of Regents)
Notes: Shared with Algebra I. Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. (PARCC)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Justify algebraically whether a function is even or odd.	Is the function $f(x) = 2x^3$ even, odd, or neither? Justify your answer algebraically.
LEVEL 4 *(meets standard)*	Given the equations or graphs of $f(x)$ and $af(x + b) + c$, where at least two of the constants a, b, and c are nonzero: • Sketch the graph $af(x + b) + c$. • Describe the transformations in words. State the values of a, b, and c. Identify even and odd functions from their graphs.	Let $f(x) = \sin x$ and $g(x) = 2 \sin (x - 4) + 3$. Describe the transformations that map $f(x)$ to $g(x)$. Is the function whose graph is shown below even, odd, or neither? 
LEVEL 3	Given the equations of $f(x)$ and the transformation $kf(x)$, $kf(x)$, or $f(x + k)$, where k is a nonzero real number, describe the single transformation in words and find the value of k.	Let $f(x) = \sin x$ and $g(x) = \sin (x) + 3$. Describe the transformations that map $f(x)$ to $g(x)$.
LEVEL 2	Describe in words the transformation that maps the function $f(x)$ to $kf(x)$, $f(x + k)$, or $f(x) + k$, where k is a nonzero real number, given their graphs. State the value of k.	Describe the transformation that maps $f(x)$ to $g(x)$, given their graphs below. $f(x)$:  $g(x)$: 
LEVEL 1	Given a graph of $f(x) = \log_a x$, $f(x) = \sin x$, or $f(x) = \cos x$, write its equation. Given the equation of one of the functions listed above, sketch its graph. State the type of symmetry contained in the graph of an even or odd function.	Graph $f(x) = \log x$.

(F-BF.B.4) Find inverse functions.

(F-BF.B.4a) Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.

Emphasis:	■□□ Additional content (19-33% of Regents)
Notes:	Shared with Algebra I.
Related:	Use logarithms to solve exponential equations (base 2, 10, *e*), evaluate logs Use logarithms to solve exponential equations (base 2, 10, *e*), evaluate logs (F-LE.A.4)

<table>
<tr><th></th><th>TASK</th><th>EXAMPLE</th></tr>
<tr><td>LEVEL 5
<i>(exceeds standard)</i></td><td>Justify algebraically that two functions are inverses of each other.</td><td>Justify algebraically that $f(x) = 2^x$ and $f(x) = \log_2 x$ are inverses of each other.</td></tr>
<tr><td>LEVEL 4
<i>(meets standard)</i></td><td>Find the inverse of a nonlinear function (if it exists) algebraically or graphically.</td><td>Find the inverse of $f(x)=\frac{x+1}{x-1}, x \neq 1$.</td></tr>
<tr><td>LEVEL 3</td><td>Find the inverse of a linear function algebraically or graphically.</td><td>Find the inverse of $f(x) = 3x - 7$.</td></tr>
<tr><td>LEVEL 2</td><td>Recognize that the inverse of a one-to-one function is formed by interchanging the domain and range.</td><td>A function has defined values that are listed in the table below.
<table>
<tr><td>x</td><td>1</td><td>3</td><td>8</td></tr>
<tr><td>y</td><td>2</td><td>–6</td><td>4</td></tr>
</table>
Which of the following tables of values represent the inverse of the function?

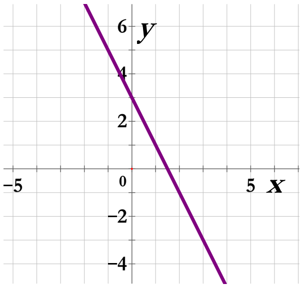
(1)
<table>
<tr><td>x</td><td>8</td><td>3</td><td>1</td></tr>
<tr><td>y</td><td>4</td><td>–6</td><td>2</td></tr>
</table>
(2)
<table>
<tr><td>x</td><td>–1</td><td>–3</td><td>–8</td></tr>
<tr><td>y</td><td>–2</td><td>6</td><td>–4</td></tr>
</table>
(3)
<table>
<tr><td>x</td><td>2</td><td>–6</td><td>4</td></tr>
<tr><td>y</td><td>1</td><td>3</td><td>8</td></tr>
</table>
(4)
<table>
<tr><td>y</td><td>4</td><td>–6</td><td>2</td></tr>
<tr><td>x</td><td>8</td><td>3</td><td>1</td></tr>
</table>
</td></tr>
<tr><td>LEVEL 1</td><td>State the definition of a function.
State the definition of an inverse of a function.</td><td>What is the definition of a function?
What is the definition of an inverse of a function?</td></tr>
</table>

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS (F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems

(F-LE.A.2) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Emphasis:	■■□ Supporting content (14-28% of Regents)
Notes:	Shared with Algebra I. Tasks will include solving multi-step problems by constructing linear and exponential functions. (PARCC)
Related:	Identify explicit and recursive sequences as functions with integer domain (F-IF.A.3) Graph and show features of graphs (F-IF.C.7) Construct linear and exponential functions, incl. arithmetic and geometric sequences (F-LE.A.2)

<table>
<tr><th></th><th>TASK</th><th>EXAMPLE</th></tr>
<tr><td>LEVEL 5
<i>(exceeds standard)</i></td><td>Describe a general method for constructing linear and exponential functions given two input-output pairs.</td><td>Describe how to write an equation for the exponential function $f(x) = ab^x$ given that it passes through the points (x_1, y_1) and (x_2, y_2).</td></tr>
<tr><td>LEVEL 4
<i>(meets standard)</i></td><td>Construct linear and exponential functions, including arithmetic and geometric sequences, given a description of a relationship or two input-output pairs.</td><td>(Aug. 2016 Alg. II Regents, #24) In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State t years after 2010?</td></tr>
<tr><td>LEVEL 3</td><td>Construct linear and exponential functions given a table of values.</td><td>Write an equation for the linear function $f(x)$ represented by the table of values below:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>…</td></tr>
<tr><td>$f(x)$</td><td>1</td><td>3</td><td>5</td><td>7</td><td>…</td></tr>
</table></td></tr>
<tr><td>LEVEL 2</td><td>Construct linear functions given their graphs or two input-output pairs.</td><td>Write an equation for the linear function $f(x)$ whose graph is shown below:

</td></tr>
<tr><td>LEVEL 1</td><td>Graph linear and exponential function given their equations.</td><td>Graph $f(x) = 5(2^x)$ for $x \ge 0$ on the coordinate plane.</td></tr>
</table>

(F-LE.A.4) For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Emphasis: ■■□ Supporting content (14-28% of Regents)
Related: Find the inverse of a function (F-BF.B.4)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Solve exponential equations with rational bases. Solve more complicated exponential equations (including those with a base that is not 2, 10, or *e*) using logarithms and the rules of exponents.	Solve for x in the equation $\left(\frac{1}{16}\right)^{x+1} = 8^x$. Solve for x in the equation $3^{3x} = 2^{2x+3}$.
LEVEL 4 *(meets standard)*	Solve an exponential equation of the form $ab^{ct} = d$, where a, b, c, and d are real numbers and $b = 2, 10$, or e.	Solve for t in the equation $9(2^{12t}) = 74$ to the nearest hundredth.
LEVEL 3	Solve an exponential equation of the form $b^{ct} = d$, where b, c, and d are real numbers and $b = 2, 10$, or e.	Solve for t in the equation $2^{12t} = 27$ to the nearest hundredth.
LEVEL 2	Using technology and the change of base formula, evaluate a logarithmic expression whose base 2.	Approximate $\log_2 42$ to the nearest hundredth.
LEVEL 1	Using technology, evaluate a logarithmic expression whose base is 10 or *e*.	Approximate log 18 to the nearest hundredth.

B. Interpret expressions for functions in terms of the situation they model

(F-LE.B.5) Interpret the parameters in a linear or exponential function in terms of a context.

Emphasis:	■□□ Additional content (19-33% of Regents)
Notes:	Shared with Algebra I. Tasks have a real-world context. Tasks are limited to exponential functions with domains not in the integers. (PARCC)
Related:	Find the inverse of a function (F-IF.B.4)

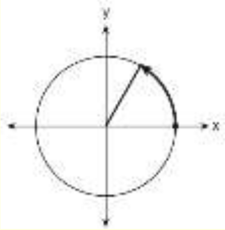
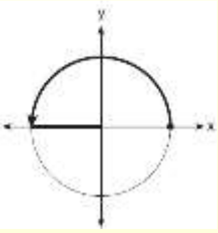
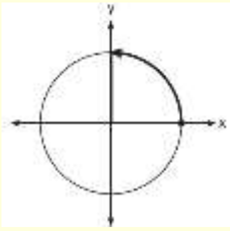
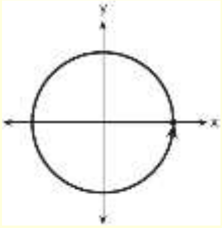
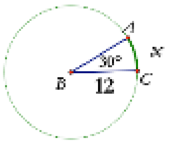
	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Interpret changes in parameters of functions in terms of a real-world context.	The breakdown of samples of two chemical compounds is represented by the functions $p(t) = 300(0.5)^t$ and $q(t) = 300(0.4)^t$, where $p(t)$ and $q(t)$ represent, respectively, the number of milligrams of each substance and t represents the time, in years. Explain what the numbers 0.4 and 0.5 represent and what they show about how the behaviors of the compounds differ.
LEVEL 4 *(meets standard)*	Identify the parameters in a linear or exponential function given its equation.	(Aug. 2016 Alg. II Regents, #An equation to represent the value of a car after t months of ownership is $v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is *not* correct? (1) The car lost approximately 19% of its value each month. (2) The car maintained approximately 98% of its value each month. (3) The value of the car when it was purchased was $32,000. (4) The value of the car 1 year after it was purchased was $25,920.
LEVEL 3	Explain the effect that a parameter of a linear or exponential function has on the function's behavior.	How does the slope of a linear function affect its behavior?
LEVEL 2	Identify the parameters in an exponential function given its equation.	If $p(t) = 300(0.5)^t$, identify the growth factor.
LEVEL 1	Identify the parameters in a linear function given its equation.	If $f(x) = -7x - 3$, find the slope and y-intercept.

TRIGONOMETRIC FUNCTIONS (F-TF)

A. Extend the domain of trigonometric functions using the unit circle

(F-TF.A.1) Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Emphasis:	■□□ Additional content (19-33% of Regents)
Related:	Use unit circle and given angles in radian measure to calculate values of 6 trig functions (F-TF.A.2) Use sine or cosine functions to model periodic behavior (F-TF.B.5) Prove Pythagorean identity and use it to find trig functions given values of other trig functions (F-TF.C.8)
Resources:	Patrick J. Eggleton, "Experiencing Radians," *Mathematics Teacher* 92 (September 1999): 468-471; "Why Radians?" The Math Forum web site, accessed February 15, 2017, http://mathforum.org/library/drmath/view/54048.html.

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain how radian measure is more naturally related than degree measure to the measures of central angles of a circle.	How does using radians instead of degrees make calculating the area of a sector easier?
LEVEL 4 *(meets standard)*	Define radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	(Aug. 2016 Alg. II, #16) Which diagram shows an angle rotation of 1 radian on the unit circle? (1)  (3)  (2)  (4) 
LEVEL 3	Given two of the following: the radius of the circle, the degree measure of the central angle, and the length of the intercepted arc, find the other quantity.	Find the length of the arc intercepted by a 60º central angle of a circle of radius 10. Circle O has radius 36 and $m\angle AOB = 60$, where A and B are points on the circle. Find the length of arc AB .
LEVEL 2	Write a proportion relating the degree measure of a central angle of a circle, 360º, the length of the intercepted arc, and the circumference.	Write an appropriate proportion to solve for x: 
LEVEL 1	Calculate the circumference of a circle given its radius.	Find in terms of π the circumference of a circle whose radius is 5.

(F-TF.A.2) Explain how the unit circle in the coordinate plane enables the extension of trigonometric function to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

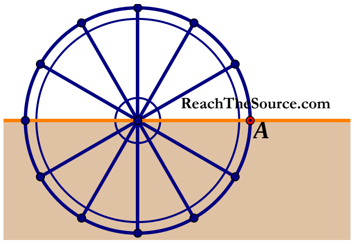
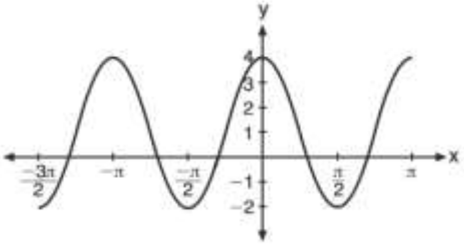
Emphasis:	■☐☐ Additional content (19-33% of Regents)
Notes:	Includes the reciprocal trigonometric functions. (NYSED)
Related:	Define radian measure (F-TF.A.1) Use sine or cosine functions to model periodic behavior (F-TF.B.5) Prove Pythagorean identity and use it to find trig functions given values of other trig functions (F-TF.C.8)
Resources:	Patrick J. Eggleton, "Experiencing Radians," *Mathematics Teacher* 92 (September 1999): 468-471; "Why Radians?" The Math Forum web site, accessed February 15, 2017, http://mathforum.org/library/drmath/view/54048.html.

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Explain how the graphs of trigonometric functions are generated from the unit circle.	Circle A has a radius of 1 and a center at the origin. Calculate tan A for all values of $A = \pi k/6$, where k is an integer from -6 to 6. Then use these values to sketch a graph of $y = \tan x$ for $0 \le x \le 2\pi$. State the equations of any asymptotes and justify your answer.
LEVEL 4 *(meets standard)*	Apply concepts of the unit circle in the coordinate plane to calculate the values of the six trigonometric functions given angles in radian measure.	Circle A has a radius of 1 and a center at the origin. If angle A is in standard position and has measure $7\pi/4$ radians, find the exact value of csc A .
LEVEL 3	Convert angle and arc measures from radians to degrees and degrees to radians.	Convert 40° to radians.
LEVEL 2	Determine angle measures, in degrees/radians, and the three basic trigonometric ratios (sin θ, cos θ, and tan θ) using the unit circle or the Pythagorean identity.	If the terminal side of angle θ in standard position passes through the point (5, 7), what is the measure of angle θ in degrees?
LEVEL 1	Calculate angle measures, in degrees, and three basic trigonometric ratios (sin θ, cos θ, and tan θ) in a right triangle.	In right triangle ABC, $m\angle B = 90$, $AB = 5$, and $BC = 7$. Express tan A as a fraction and find $m\angle A$ to the nearest tenth of a degree.

B. Model periodic phenomena with trigonometric functions

(F-TF.B.5) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Transform functions, recognize even and odd functions (F-BF.B.3)
Use unit circle and given angles in radian measure to calculate values of 6 trig functions (F-TF.A.2)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Create appropriate trigonometric functions to model periodic phenomena based on a verbal description of the amplitude, frequency, and midline.	(Algebra II Module 2, EngageNY, p. 217) An amusement park has a small Ferris wheel, called a kiddie wheel, for toddlers. The points on the circle in the diagram represent the position of the cars on the wheel. The kiddie wheel has four cars, makes one revolution every minute, and has a diameter of 20 feet. The distance from the ground to a car at the lowest point is 5 feet. Assume $t = 0$ corresponds to a time when car 1 is closest to the ground. Find a formula for a function that models the height of car 1 with respect to time as the kiddie wheel rotates.
LEVEL 4 *(meets standard)*	Construct an appropriate trigonometric function to model periodic phenomena by correctly interpreting amplitude, frequency and midline.	(Jun. 2016 Alg. II, #24) The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Write the equation of a function $V(t)$ that represents the value of the voltage as it flows through the electric wires, where t is time in seconds.
LEVEL 3	Choose an appropriate trigonometric function to model periodic phenomena by correctly interpreting amplitude, frequency, or midline.	(Jun. 2016 Alg. II, #24) In most households, the maximum voltage is 120 volts, and there are 60 cycles *every second*. The value of the voltage as it flows through the electric wires, where t is time in seconds, can be modeled by the function $V(t) = 120\sin(bt)$. Find the value of b.
LEVEL 2	Given a situation, determine the appropriate trigonometric function that best represents the model.	A water wheel with a diameter of 2 meters rotates counterclockwise. The water wheel is located in a channel such that the lower half of the wheel is below ground level, as shown in the accompanying diagram. A point at ground level on the wheel (marked A on the diagram) is located at the 3 o'clock position when the wheel begins to rotate. The wheel makes one revolution per minute. What trigonometric function is best used to model the height of point A with respect to the ground as the wheel rotates? 
LEVEL 1	Given a graph, identify which trigonometric function is being modeled. Identify amplitude, frequency or midline of a given trigonometric model.	Identify which trigonometric function is shown in the accompanying graph. State its amplitude. (Image credit: Jan. 2015 Alg. 2/Trig, #38) 

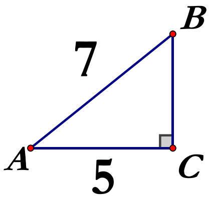
C. Prove and apply trigonometric identities

(F-TF.C.8) Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Emphasis: ■□□ Additional content (19-33% of Regents)

Related: Prove and use polynomial identities (A-APR.C.4)

Use unit circle and given angles in radian measure to calculate values of 6 trig functions Use unit circle and given angles in radian measure to calculate values of 6 trig functions (F-TF.A.2)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Use the Pythagorean, quotient, and reciprocal identities to prove trigonometric identities.	Prove that $\tan x + \cot x = \csc x \sec x$.
LEVEL 4 *(meets standard)*	Prove the identity $\sin^2\theta + \cos^2\theta = 1$. Find the value of any of the six trigonometric functions given any other trigonometric function value and appropriate information about its quadrant.	Let θ be an angle in standard position on a unit circle. Prove the identity $\sin^2\theta + \cos^2\theta = 1$. If $\tan A = \frac{\sqrt{7}}{3}$ and $\sin A < 0$, find $\cos A$.
LEVEL 3	Given a point on a circle centered at the origin and three of the following, find the missing quantity: radius of circle, x-coordinate of the point, y-coordinate of the point, quadrant in which the point is located.	The point $(5, y)$ lies in Quadrant IV on circle O, which is centered at the origin and has radius 12. Find the value of y.
LEVEL 2	Given the signs of trigonometric functions of an angle in standard position, identity the quadrant in which the terminal side of the angle is located.	Let θ be an angle in standard position on a unit circle. If $\sin\theta > 0$ and $\tan\theta < 0$, in what quadrant is the terminal side of θ located?
LEVEL 1	Use the Pythagorean Theorem to solve for a missing side of a right triangle in simplest radical form. Verify that $\sin^2\theta + \cos^2\theta = 1$ for a given value of θ.	In right triangle ABC, $\angle C$ is a right angle, $AB = 7$, and $AC = 5$, as shown in the accompanying diagram. Find BC.  Show that the equation $\sin^2\theta + \cos^2\theta = 1$ is true when $\theta = 30°$.

STATISTICS AND PROBABILITY (14%-21% OF REGENTS EXAM)

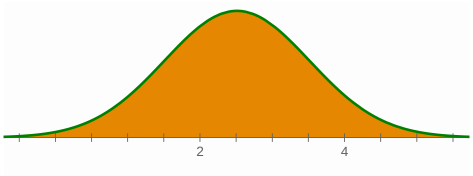
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA (S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable

(S-ID.A.4) Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Emphasis: ■□□ Additional content (19-33% of Regents)

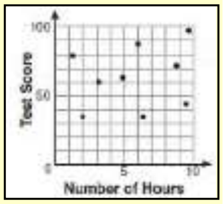
Related: Determine if a statistic is likely to occur based on a given simulation (S-IC.A.2)
Given simulation model based on sample, construct 95% interval centered on sample; determine if suggested parameter is plausible (S-IC.B.4)

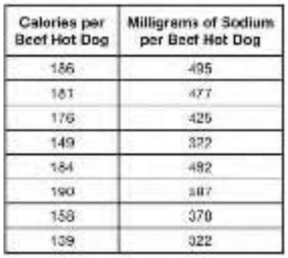
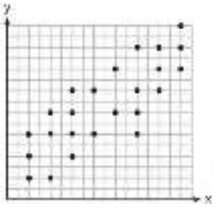
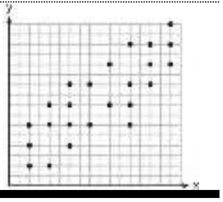
	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine whether a real-world situation may fit a normal distribution. Use information from a sample to determine if the distribution of the population is approximately normal.	A random sample from a data set has the following values: 22, 17, 18, 29, 22, 23, 24, 23, 17, 21. Is the data set normal?
LEVEL 4 *(meets standard)*	Calculate and interpret the population percentage for normally distributed data.	The length of time employees have worked at a corporation is normally distributed, with a mean of 16.2 years and standard deviation of 1.4 years. In a company cutback, the lowest 10% in seniority are laid off. What is the maximum length of time an employee could have worked and still be laid off?
LEVEL 3	Sketch a normal distribution model given the mean and standard deviation for a set of data.	A normally distributed data set has a mean of 16 and a standard deviation of 1.2. Sketch a labeled normal curve for the data
LEVEL 2	Identify the mean and standard deviation given a normal distribution and calculate a z-score for a given set of data.	Determine the mean and standard deviation of the normal distribution shown below: 
LEVEL 1	State the characteristics of the normal curve.	What are the characteristics of the normal curve?

B. Summarize, represent, and interpret data on two categorical and quantitative variables

(S-ID.B.6) Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
(S-ID.B.6a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Emphasis: ■■□ Supporting content (14-28% of Regents)
Notes: Shared with Algebra I. Tasks have a real world context. Tasks are limited to exponential functions with domains not in the integers and trigonometric functions. (NYSED)

<table>
<tr><th></th><th>TASK</th><th>EXAMPLE</th></tr>
<tr><td>LEVEL 5
<i>(exceeds standard)</i></td><td>Make inferences about data based on scatter plots and functions that fit data set.</td><td>A teacher surveyed the number of hours of television that students watched the week before the final exam and the final exam score. The teacher plotted the data on a scatterplot, shown at right. The teacher concludes that students who watched more television generally received lower scores. Is this conclusion justified? Explain.

</td></tr>
<tr><td>LEVEL 4
<i>(meets standard)</i></td><td>Explain in context the relationship between the variables.
Fit an exponential or trigonometric model to a given data set.
Write a regression equation and use it to solve problems in context.</td><td>(Jan. 2017 Alg. II Regents, #13) The price of a postage stamp in the years since the end of World War I is shown in the scatterplot at right. The equation that best models the price, in cents, of a postage stamp based on these data is

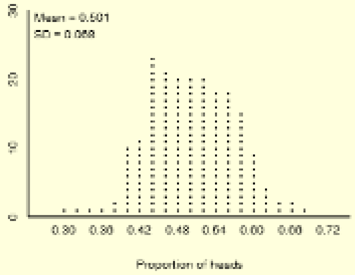

(1) $y = 0.59x - 14.82$ (3) $y = 1.43(1.04)^x$
(2) $y = 1.04(1.43)^x$ (4) $y = 24\sin(14x) + 25$</td></tr>
<tr><td>LEVEL 3</td><td>Represent quantitative variables on a scatter plot.
Given a function that fits a data set, interpret an input and output value in context. Predict an output value for a given input.</td><td>A nutritionist collected information about different brands of beef hot dogs. She made a table, shown here, showing the number of Calories and the amount of sodium in each hot dog. Construct a scatter plot for the data.

<table>
<tr><th>Calories per Beef Hot Dog</th><th>Milligrams of Sodium per Beef Hot Dog</th></tr>
<tr><td>156</td><td>495</td></tr>
<tr><td>181</td><td>477</td></tr>
<tr><td>176</td><td>425</td></tr>
<tr><td>149</td><td>322</td></tr>
<tr><td>184</td><td>482</td></tr>
<tr><td>190</td><td>587</td></tr>
<tr><td>158</td><td>370</td></tr>
<tr><td>139</td><td>322</td></tr>
</table></td></tr>
<tr><td>LEVEL 2</td><td>Given a scatterplot for a data set, determine the direction of the association.</td><td>(Aug. 2012 Int. Alg. Regents, #4) The scatter plot shown represents a relationship between <i>x</i> and <i>y</i>. This type of relationship is
(1) a positive correlation
(2) a negative correlation
(3) a zero correlation
(4) not able to be determined

</td></tr>
<tr><td>LEVEL 1</td><td>Given a scatter plot for a data set, determine whether the association is linear, quadratic, or exponential.</td><td>The scatter plot shown below represents a relationship between <i>x</i> and <i>y</i>. Is this type of relationship linear, quadratic, or exponential?

</td></tr>
</table>

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS (S-IC)

A. Understand and evaluate random processes underlying statistical experiments

(S-IC.A.1) Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
(S-IC.A.2) Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Emphasis: ■■☐ Supporting content (14-28% of Regents)
Related: Given simulation model based on sample, construct 95% interval centered on sample; determine if suggested parameter is plausible (S-IC.B.4)
Compare two treatments and determine if the difference between parameters is significant (S-IC.B.5)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Calculate the mean and standard deviation of a sampling distribution and use that information to determine if a value for a sample proportion or sample mean is likely to occur.	A baseball team in a large city believes that 55% of the city's population supports a new stadium for the team. The team conducts a survey of a random sample of 500 city residents and finds that 260 of them support a new stadium. Does the survey's result indicate that the city supports a new stadium for the team? Explain.
LEVEL 4 *(meets standard)*	Determine if a value for a sample proportion or sample mean is likely to occur based on a given simulation.	A football league wants to test a coin to be used to decide who controls the ball first in games. It flips the coin 50 times and the proportion of heads is recorded. This is repeated 200 times. A dotplot of the 200 sample proportions is shown. The mean of the distribution is 0.501 and the standard deviation is 0.068. Does the coin appear to be fair? Explain. 
LEVEL 3	Generate a simulation model for a given real-world situation.	A football league wants to test a coin to be used to decide who controls the ball first in games. It flips the coin 50 times and the proportion of heads is recorded. Create a histogram that shows the frequency distribution of 200 coin flips.
LEVEL 2	Calculate a sample mean or sample proportion from given data.	A football league wants to test a coin to be used to decide who controls the ball first in games. It flips the coin 50 times and finds that 28 of them came up heads. Calculate the appropriate statistic from this sample.
LEVEL 1	Identify a sample mean or a sample proportion.	A football league wants to test a coin to be used to decide who controls the ball first in games. It flips the coin 50 times and finds that 40% came up heads. Is the 40% a sample mean or a sample proportion?

B. Understand and evaluate random processes underlying statistical experiments

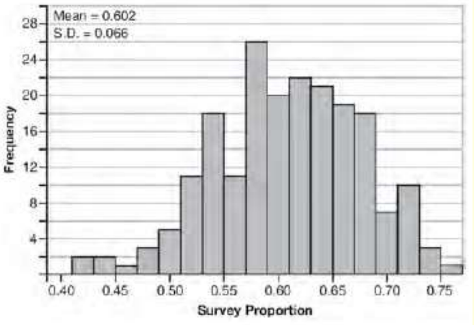
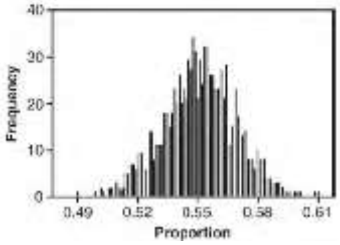
(S-IC.B.3) Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Emphasis: ■■■ Major content (51-65% of Regents)
Related: Use statistical language to draw conclusions from numerical summaries (S-IC.B.6a) and critique claims (S-IC.B.6b)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Compare and contrast the purposes and differences among sample surveys, experiments, and observational studies. Make inferences and justify conclusions based on appropriate data collection methods.	When would observational studies be more appropriate than controlled experiments?
LEVEL 4 *(meets standard)*	Explain how randomization is accomplished in sample surveys, experiments, and observational studies and describe the purpose of each.	What is the difference between a survey and an experiment? A toothpaste company wants to determine the effect of an added ingredient to its toothpaste. It has found 500 adult volunteers who are willing to participate in a study conducted by the company. Describe an appropriate method of conducting an experiment.
LEVEL 3	Identify which method of data collection is appropriate to a given context.	A medical research facility wants to determine if chewing tobacco increases the risk of cancer. Which method of data collection would be most appropriate?
LEVEL 2	Describe how randomization affects a sample.	The librarian of a large school wants to determine who uses the library's computers to play games instead of do schoolwork. She wants to survey a sample of students who use the library computers. Explain the benefits of making her sample random.
LEVEL 1	Define each type of data collection.	What is an experiment?

(S-IC.B.4) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Emphasis: ■■■ Major content (51-65% of Regents)
Related: Determine if a statistic is likely to occur based on a given simulation (S-IC.A.2)
Use statistical language to draw conclusions from numerical summaries (S-IC.B.6a) and critique claims (S-IC.B.6b)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Construct and interpret a 95% confidence interval for a parameter given a statistic and a sample size.	In a recent poll of 1,051 randomly selected adults aged 18 and older, 58% of respondents surveyed said that they spend too much time on cell phones or smartphones. Calculate and interpret the margin of error with 95% confidence.
LEVEL 4 *(meets standard)*	Given the mean and a standard deviation of a simulation model, construct a 95% confidence interval for a parameter and determine if a suggested parameter is plausible.	(Jun. 2016 Alg. II Regents, #35) Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below. Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50%-50% split. Explain what statistical evidence supports this concern. 
LEVEL 3	Given a simulation model, estimate the statistic and the margin of error for a 95% confidence interval	(Aug. 2016 Alg. II Regents, #12) A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the accompanying diagram. Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to  (1) 0.01 (2) 0.03 (3) 0.06 (4) 0.12
LEVEL 2	Given a 95% confidence interval, state the statistic used and the margin of error.	Determine the statistic and the margin of error used to construct a 95% confidence interval of (0.72, 0.78).
LEVEL 1	Define a confidence interval.	What is a confidence interval?

(S-IC.B.5) Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Emphasis: ■■■ Major content (51-65% of Regents)

Related: Determine if a statistic is likely to occur based on a given simulation (S-IC.A.2)

(S-IC.B.6) Evaluate reports based on data.

Emphasis: ■■■ Major content (51-65% of Regents)

Related: Determine if a statistic is likely to occur based on a given simulation (S-IC.A.2)
Understand uses of, relationship of randomization to, and differences between surveys, experiments, observational studies (S-IC.B.3)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Determine the effect of bias on the results of a statistical study.	*The Literary Digest* was an influential weekly news magazine published by Funk & Wagnalls. Before the 1936 election, the magazine polled 10 million individuals, of whom about 2.4 million responded, regarding the likely outcome of the presidential election. The *Literary Digest* surveyed individuals from the following lists: its own readers, registered automobile owners, and telephone users. The poll showed that the Republican candidate, Gov. Alf Landon of Kansas, would be the overwhelming winner. Explain how the *Literary Digest*'s sampling method skewed the results of its study.
LEVEL 4 *(meets standard)*	Critique claims from informational texts.	In 2015, the New York Mets and the Kansas City Royals played in Major League Baseball's World Series. A newspaper in upstate New York conducted an online poll in which readers were invited to pick the team that would win the World Series. 65% of respondents said that the Mets would win. The newspaper concluded that the nation was rooting for the Mets, who have not won a championship since 1986, to win. Comment on the newspaper's conclusion.
LEVEL 3	Identify potential sources of bias in statistical studies. Identify the statistical evidence needed to evaluate a claim.	To determine public interest in increasing funding for public parks, a large city decides to randomly survey people in 10 public playgrounds around the city. Identify potential sources of bias in the survey.
LEVEL 2	Differentiate between bias and sampling variability.	A candy manufacturing company wants to see if one of its cutting machines is accurately cutting similarly sized rectangular bars of chocolate into squares that are two inches long on each side. Every fifth square that is cut from the machine is measured. A sample of 10 squares yields measurements of 2.07, 2.02, 1.96, 2.01, 1.93, 2.01, 2.09, 2.03, 1.99, and 1.98 inches. None of the measurements are two inches. Is this because the measurements were probably biased, or are the differences likely due to sampling variability?
LEVEL 1	Define bias. Define sampling variability.	What is bias?

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY (S-CP)

A. Understand independence and conditional probability and use them to interpret data

(S-CP.A.1) Describe events as a subset of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.

Emphasis:	■☐☐ Additional content (19-33% of Regents)
Related:	Construct, interpret, use two-way tables to determine if events are independent (S-CP.A.4) Use Addition Rule of probability and interpret the answer (S-CP.B.7)

	TASK	EXAMPLE
LEVEL 5 *(exceeds standard)*	Prove theorems about rules of inference that relate to unions, intersections, or complements.	Prove that the complement of the intersection of two sets is equal to the union of the complements of the sets, i.e. $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
LEVEL 4 *(meets standard)*	Describe in context the union and intersection of sets or the complement of a set.	List all whole numbers from 1 to 20 that are even but not multiples of 3.
LEVEL 3	Given three or more sets whose elements are listed, describe their union or intersection.	Let A = {1, 2, 3, 4, 5}, B = {2, 4, 6, 7, 8}, and C = {3, 5, 6, 9, 10, 11}. List all elements in the union of A and B.
LEVEL 2	Given two sets whose elements are listed, describe their union or intersection. Given a set whose elements are listed, describe its complement.	Let A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 7, 8}. What is the union of A and B?
LEVEL 1	Define the union and intersection of sets. Define the complement of a set. Define a sample space.	What is the union of two sets?

(S-CP.A.2) Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
(S-CP.A.3) Understand the conditional probability of A given B as P(A and B)/P(B) and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
(S-CP.A.4) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
(S-CP.A.5) Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Describe events as subsets of sample space or unions, complements, intersections of other events (S-CP.A.1)
Use Addition Rule of probability and interpret the answer (S-CP.B.7)

<table>
<tr><th></th><th>TASK</th><th>EXAMPLE</th></tr>
<tr><td>LEVEL 5
<i>(exceeds standard)</i></td><td>Explain the relationship between mutually exclusive and independent events.</td><td>Why must mutually exclusive events be dependent?</td></tr>
<tr><td>LEVEL 4
<i>(meets standard)</i></td><td>Construct a two-way table based on a real-world situation.
Given a real-world situation, determine if two events are independent.</td><td>In a large community, 75% of the people are adults, 80% of the people have traveled outside the state, and 15% are adults who have not traveled outside the state. Determine whether being an adult is independent of traveling outside the state.</td></tr>
<tr><td>LEVEL 3</td><td>Given a two-way table representing a real-world situation, interpret a cell in context.
Given a two-way frequency table, construct a relative frequency table.
Use the formula $P(A) \bullet P(B|A) = P(A \cap B)$ to calculate conditional probability.</td><td>To gauge public opinion on whether a new high school building should be built, a town school board conducts a survey of 515 residents. The board wants to determine if men and women have significantly different opinions about the new construction. The data is summarized below. Construct a relative frequency table for the data.
<table>
<tr><th></th><th></th><th colspan="4">SHOULD OUR TOWN BUILD A NEW H.S.?</th></tr>
<tr><th></th><th></th><th>Yes</th><th>No</th><th>No Answer</th><th>TOTAL</th></tr>
<tr><td>GENDER</td><td>Male</td><td>119</td><td>116</td><td>6</td><td>241</td></tr>
<tr><td></td><td>Female</td><td>154</td><td>114</td><td>6</td><td>274</td></tr>
<tr><td></td><td>TOTAL</td><td>273</td><td>230</td><td>12</td><td>515</td></tr>
</table></td></tr>
<tr><td>LEVEL 2</td><td>Given a completed two-way table for events A and B, identify $P(A)$, $P(B)$, $P(A \text{ and } B)$, $P(B|A)$, and $P(A|B)$.
Given two events A and B and $P(A)$, $P(B)$, and $P(A \text{ and } B)$, determine if A and B are independent.</td><td>Given A and B such that $P(A) = 0.4$, $P(B) = 0.3$, and $P(A \text{ and } B)$, determine if A and B are independent.</td></tr>
<tr><td>LEVEL 1</td><td>Complete a partially completed two-way relative frequency table.</td><td>Complete the following two-way table.
<table>
<tr><th></th><th></th><th colspan="3">EVENT A</th></tr>
<tr><th></th><th></th><th>Yes</th><th>No</th><th>TOTAL</th></tr>
<tr><td>EVENT B</td><td>Yes</td><td>0.15</td><td></td><td>0.38</td></tr>
<tr><td></td><td>No</td><td></td><td></td><td>0.62</td></tr>
<tr><td></td><td>TOTAL</td><td>0.32</td><td>0.68</td><td>1.00</td></tr>
</table></td></tr>
</table>

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model

(S-CP.B.6) Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Construct, interpret, use two-way tables to determine if events are independent (S-CP.A.4)

(S-CP.B.7) Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Emphasis: ■□□ Additional content (19-33% of Regents)
Related: Describe events as subsets of sample space or unions, complements, intersections of other events (S-CP.A.1)
Construct, interpret, use two-way tables to determine if events are independent (S-CP.A.4)

<table>
<tr><th></th><th>TASK</th><th>EXAMPLE</th></tr>
<tr><td>LEVEL 5
<i>(exceeds standard)</i></td><td>Calculate compound probability involving three or more events.</td><td>(Aug. 2004 Math A Regents, #19) Seventy-eight students participate in one or more of three sports: baseball, tennis, and golf. Four students participate in all three sports; five play both baseball and golf, only; two play both tennis and golf, only; and three play both baseball and tennis, only. Seven students play only tennis and one plays only golf. If one of the 78 students is randomly selected, what is probability that the student plays only baseball?</td></tr>
<tr><td>LEVEL 4
<i>(meets standard)</i></td><td>Given a real-world situation, calculate and interpret the probability of a compound event using the Addition Rule.</td><td>(Jun. 2016 Alg. II Regents, #29) A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is 974/1376, what is the probability that a student participates in both sports and music?</td></tr>
<tr><td>LEVEL 3</td><td>Given a real-world situation with information about sets A and B, find the number of elements in the union or intersection of the sets.</td><td>(Aug. 2006 Math A Regents, #31) In Clark Middle School, there are 60 students in seventh grade. If 25 of these students take art only, 18 take music only, and 9 do not take either art or music, how many take both art and music?</td></tr>
<tr><td>LEVEL 2</td><td>Given a completed two-way table for events A and B, identify $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A \text{ or } B)$.</td><td>Given the following two-way table containing the probabilities of two events A and B, find $P(A \text{ and } B)$.
<table>
<tr><th></th><th></th><th colspan="3">EVENT A</th></tr>
<tr><th></th><th></th><th>Yes</th><th>No</th><th>TOTAL</th></tr>
<tr><td rowspan="3">EVENT B</td><td>Yes</td><td>0.15</td><td>0.23</td><td>0.38</td></tr>
<tr><td>No</td><td>0.17</td><td>0.45</td><td>0.62</td></tr>
<tr><td>TOTAL</td><td>0.32</td><td>0.68</td><td>1.00</td></tr>
</table></td></tr>
<tr><td>LEVEL 1</td><td>Given three of the probabilities in the Addition Rule, find the missing probability.</td><td>Given two events A and B such that $P(A) = 0.67$, $P(B) = 0.52$, $P(A \text{ and } B) = 0.46$, find $P(A \text{ or } B)$.</td></tr>
</table>